TG 547
JANUARY 1964

ANALYSIS of SATELLITE TIME DATA

by

W. H. GUIER

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
SILVER SPRING, MARYLAND

Operating under Contract NOw 62-0604-c with the Bureau of Naval Weapons, Department of the Navy

(14) Rept. no. TG-547 9
(11) JAN [redacted] 64,
COPY NO. 35

(6) ANALYSIS of SATELLITE TIME DATA,

(10) by

W. H. GUIER.

(3)

~~THE [redacted] HOPKINS UNIVERSITY~~
(5) APPLIED PHYSICS LAB[redacted], Johns Hopkins Univ., Silver Spring, Md.

The Johns Hopkins University
APPLIED PHYSICS LABORATORY
Silver Spring, Maryland

ANALYSIS OF SATELLITE TIME DATA

W. H. Guier

omicron alpha eta beta mu

Satellites 1961 $o1$, 1961 $\alpha\eta 1$, and 1962 $\beta\mu 1$ are instrumented to supply time data as well as doppler data. Satellite time markers are generated in these satellites by "counting down" the satellite's stable oscillator and producing a unique modulating on the doppler carriers each time a fixed number of oscillator cycles have occurred. The TRANET doppler receiving stations are equipped to recognize this modulation and to automatically punch (on the same paper tape that doppler data is punched) the station's time (WWV) at which this special modulation is received. After appropriate calibration, this time data can serve as a secondary time standard. This report presents the information required to recognize and process satellite time data and to determine the WWV time at which the satellites transmitted satellite time markers.

The report is divided into two sections. The first section presents the basic equations that connect the satellite transmitter frequency, the station's satellite time data, and the times that the satellites transmitted the time markers. The second section presents the information required to recognize, re-format, and reduce the experimental satellite time data to a form appropriate to the theory presented in the first section.

---

*In this report, WWV Time is used to designate WWV's estimate of UT2.

I. DERIVATION OF EQUATIONS FOR SATELLITE TIME

This section presents the equations connecting the satellite's oscillator frequency, the stations' data when the satellite transmitted time markers, and inferred values for the times when time markers are transmitted relative to a chosen standard of time. The section is divided into three sub-sections. The first sub-section presents the notation for important quantities which are used throughout this report. The second sub-section presents the equations to determine the absolute frequency of the satellite as a function of time. The third sub-section presents one method for statistically inferring when the satellite transmitted time markers. Also included in the third section is a suggestion for a simplified equation to predict into the future the times that satellite time markers are transmitted for station alert purposes.

A. Notation

1. Satellite Frequency*

f_R = satellite reference frequency (mHz);

$\Delta\upsilon_S$ = standard offset of satellite frequency (hz/mHz);

$f_{S,O}$ = standard satellite transmitter frequency (hz),

$= f_R(10^6 + \Delta\upsilon_S)$;

---

*1 Hz = 1 cps., and 1 mHz = 1 mc.

$f_S(t)$ = estimated satellite frequency as function of time (Hz);

$\delta f_S(t)$ = estimated deviation of satellite frequency from standard (Hz),

$= f_S(t) - f_{S,0}$;

$\{u_\ell(t,t_0)\}$ = functions chosen for expansion of $\delta f_S(t)$,

$$\delta f_S(t) = \sum_{\ell=1}^{n_f} a_{f,\ell}\, u_\ell(t,t_0);$$

t_0 = epoch of satellite frequency equation,

$\{a_{f,\ell}\}$ = expansion coefficients of deviation of satellite frequency for the $\{u_\ell(t,t_0)\}$;

$$v_\ell(t,t_0) = \int_{t_0}^{t} dt'\, u_\ell(t',t_0);$$

δf_γ = experimental determination of the deviation of the satellite frequency from its standard, scaled to f_R, for the γ-th pass, (Hz);

$t_{c\gamma}$ = time of closest approach of the satellite to receiving station for the γ-th pass, (sec).

2. Satellite Time

$t_0^{(s)}$ = epoch of satellite time equation;

$N_S(t)$ = number of cycles of satellite oscillator (scaled to f_R) that have occurred after $t_0^{(s)}$;

$N_{C,S}$ = number of cycles of satellite oscillator (scaled to f_R) that occur between two successive transmissions of time markers;

K = number of satellite time markers that have been transmitted since $t_0^{(s)}$;

t_k = inferred time of K^{th} time marker transmission;

$t_k^{(E)}$ = experimentally measured time of K^{th} time marker transmission;

Int. $[X(t)]$ = rounded integer part of the number $X(t)$.

B. Inferred Satellite Frequency as a Function of Time

Within a few days after successful launch of a satellite containing stable oscillators, a standard offset of the oscillator frequency from exactly one mHz is chosen. This standard offset, $\Delta\nu_S$, normally remains fixed for the useful life of the satellite. Table I gives the value of $\Delta\nu_S$ for those two satellites currently in orbit which transmit time markers.

Satellite	(hz/mHz
1961 οl	-35.333333
1962 βμl	-77.000000

TABLE I. Standard Offset of Satellite Oscillators

An accurate experimental measurement of the satellite oscillator frequency can be obtained during pre-processing of the doppler data by adjusting the receiving station's position to provide the best fit to the experimental doppler data.(1) This experimental measurement of the satellite oscillator frequency for the γ-th pass is usually scaled to some convenient reference frequency, f_R, and is given as a correction term, δf_γ, to the standard satellite frequency, $f_{S,0} = f_R(10^6 + \Delta\nu_S)$.

Consequently, letting the time of closest approach for the γ-th pass, t_{c_γ}, be the time associated with the experimental measurement of frequency, δf_γ, it can be seen that the deviation of the satellite frequency from its standard frequency, $\delta f_S(t)$, can be directly inferred from the experimental data, $\{\delta f_\gamma\}$. Let the deviation of the satellite frequency be parameterized in the form:

$$\delta f_S(t) = \sum_{\ell=1}^{n_f} a_{f,\ell}\, u_\ell(t,t_0) \qquad (1.1)$$

where the $u_\ell(t,t_0)$ are known functions of the time and chosen to accurately represent the variations of the satellite oscillator frequency as a function of the time $\underline{t}$, and the frequency epoch, $\underline{t_0}$.*
Choosing a convenient epoch; t_0, for equation (1.1), a least squares determination for the expansion coefficients, $\{a_{f,\ell}\}$:

---

* 1961 $\alpha\Pi$1 data indicates that for active temperature controlled satellite oscillators, $u_1 = 1$ and $u_2\,(t,t_0) = t - t_0$ are adequate for parameterizing the variations of the satellite frequency, so long as the ambient satellite temperature remains within the control range.

(1)"Orbit Improvement Program", APL/JHU Report TG-401.

$$F(\vec{a}_f) = \frac{1}{n_p} \sum_{\gamma=1}^{n_p} w_\gamma \left[\sum_{\ell=1}^{n_f} a_{f,\ell}\, u_\ell(t_{c\gamma}, t_0) - \delta f_\gamma \right]^2 , \qquad (1.2)$$

$$\frac{\partial F(\vec{a}_f)}{\partial a_{f,\ell}} = 0; \quad \ell = 1,2,\ldots n_f,$$

where n_p = total number of satellite passes,

provides a method for inferring the satellite oscillator frequency at any desired time. The statistical weighting factors, w_γ, should be chosen, to correspond to an a priori estimate of the quality of the doppler data and the accuracy of the satellite orbit used to infer the experimental $\{\delta f_\gamma\}$.

C. Inferred Times when the Satellite Transmits Time Markers

The satellites considered in this report initiate the transmissions of time markers by counting the number of cycles generated by the satellite oscillator (positive zero crossings of the satellite oscillator voltage) and triggering the transmission each time that the count reaches a predetermined integer number. Consequently, except for a scaling factor, the oscillator frequency represents the satellite 'clock rate' and all that is required to determine accurately the times that markers are transmitted is to, 1) determine accurately the satellite frequency, and 2) determine one time of transmission of a marker relative to some accepted standard of time (the epoch of the time equation). This section presents the information required to determine the scaling factor for the clock rate and the equations to determine the time of transmission of one time marker, given experimental measurements of the times of transmission of time markers relative to an accepted time standard. In addition, this section presents the equations necessary to check the station clock of any receiving station. Finally, this section presents one form of a simplified equation that is convenient to use for extrapolating marker transmission times into the future to an accuracy sufficient for station alert purposes.

If $N_S(t)$ is the number of cycles that the satellite oscillator (scaled to f_R) has generated since some epoch, then the change in $N_S(t)$ over a short time interval can be used as a measure of the satellite frequency with sufficient accuracy.

$$\frac{N_S(t + \Delta t) - N_S(t)}{\Delta t} = f_S(t) = f_{S,0} + \delta f_S(t), \tag{1.3}$$

or, to sufficient accuracy, $N_S(t)$ can be represented by:

$$N_S(t) = N_S(t_0) + f_{S,0}(t - t_0) + \int_{t_0}^{t} dt' \ \delta f_S(t'). \tag{1.4}$$

Let $N_{C,S}$ be the number of cycles generated by the oscillator between two successive time marker transmissions, (again scaled to f_R) and let

$$K_S(t) \equiv \frac{N_S(t)}{N_{C,S}} = K_S(t_0) + \frac{f_{S,0}}{N_{C,S}}(t - t_0) + \frac{1}{N_{C,S}} \int_{t_0}^{t} dt' \ \delta f_S(t') \tag{1.5}$$

Finally, if the epoch for $N_S(t)$ is the time, $t_0^{(s)}$, that a marker was transmitted, the time that the Kth time marker is transmitted, t_k, is determined by the equation:

$$\text{Integer } K = K_S(t_k)$$

$$= K_S(t_0) + \frac{f_{S,0}}{N_{C,S}} (t_k - t_0) + \frac{1}{N_{C,S}} \int_{t_0}^{t_k} dt' \, \delta f_S(t'), \tag{1.6A}$$

with the boundary condition that:

$$0 = K_S(t_0) + \frac{f_{S,0}}{N_{C,S}} (t_0^{(s)} - t_0) + \frac{1}{N_{C,S}} \int_{t_0}^{t_0^{(s)}} dt' \, \delta f_S(t'). \tag{1.6B}$$

Combining equations (1.6),

$$K = \frac{f_{S,0}}{N_{C,S}} (t_k - t_0^{(s)})$$

$$+ \frac{1}{N_{C,S}} \left[\int_{t_0}^{t_k} dt' \, \delta f_S(t') - \int_{t_0}^{t_0^{(s)}} dt' \, \delta f_S(t') \right], \tag{1.7}$$

where K is always an integer.

Finally, substituting equation (1.1) of the previous sub-section into equation (1.7), and letting:

$$v_\ell(t,t_0) = \int_{t_0}^{t} dt' \, u_\ell(t,t_0), \tag{1.8A}$$

then

$$K = \frac{f_{S,0}}{N_{C,S}} (t_k - t_0^{(s)}) + \frac{1}{N_{C,S}} \sum_{\ell=1}^{n_f} a_{f,\ell}[v_\ell(t_k,t_0) - v_\ell(t_0^{(s)},t_0)] \qquad (1.8B)$$

Table I in Section I.A. listed values of $\Delta\nu_S$ from which $f_{S,0}$ can be calculated. Table II, given below, lists the values of $N_{C,S}$ for the three satellites of interest in this report. Assuming that the frequency epoch, t_0, has been chosen and the values of the frequency expansion coefficients, $a_{f,\ell}$, have been inferred from doppler data as discussed in section I.A., there remain in equations (1.8) only three possible unknowns: K, t_k, or $t_0^{(s)}$. It can be seen that if any two of these three are known, the third can be found from the above equations. Three cases are now discussed, the case number depending upon which of the three variables is considered unknown.

Satellite	$N_{C,S}$ at 3 mHz.
1961 o1	100712448
1961 $\alpha\tau$1	67092480
1962 $\beta\mu$1	268435456

TABLE II. Number of Oscillator Cycles Occurring between Time Markers

Case 1: The Number of the Time Marker, K, is Unknown.

From the numbers given in Tables I and II, it can be seen that two time markers can not occur more frequently than about every 20 seconds. Consequently, given an approximate time of transmission of a time marker, $t_k^{(E)}$, and an approximate value for the time of the zeroth time marker transmission, $t_0^{(s)}$, (epoch for time equation), both accurate to within a few seconds, the value of the integer K corresponding to $t_k^{(E)}$ is uniquely given by:

$$K = \mathrm{Int}\left[\frac{f_{S,0}}{N_{C,S}}\left(t_k^{(E)} - t_0^{(s)}\right) + \frac{1}{N_{C,S}} \sum_{\ell=1}^{n_f} a_{f,\ell}\left(v_\ell(t_k^{(E)},t_0) - v_\ell(t_o^{(s)},t_o)\right)\right], \tag{1.9}$$

where Int [ ] represents the rounded integer part of the quantity contained in the bracket.

Case 2: The Inferred Time, t_k, is Unknown.

Given an accurate value of the epoch, $t_0^{(s)}$, and an approximate value for the time of transmission of a time marker, $t_k^{(E)}$, the corresponding integer K can be found from equation (1.9). Then, the inferred value, t_k, which lies closest to $t_k^{(E)}$ can be found from the equation:

$$t_k = t_o^{(s)} + \frac{N_{C,S}}{f_{S,0}} \cdot K - \frac{1}{f_{S,0}} \sum_{\ell=1}^{n_f} a_{f,\ell} \left[v_\ell(t_k,t_0) - v_\ell(t_0^{(s)},t_0)\right] \tag{1.10A}$$

This is a transcendental equation in the unknown t_k. However, the contribution of the sum to equation (1.10A) is so small that an iterative procedure converges very rapidly (about two iterations). Let $t_k^{(m)}$ be the result after m iterations. Then:

$$t_k^{(m+1)} = t_0^{(s)} + \frac{N_{C,S}}{f_{S,0}} \cdot K - \frac{1}{f_{S,0}} \sum_{\ell=1}^{n_f} a_{f,\ell}\left[v_\ell(t_k^{(m)},t_0) - v_\ell(t_0^{(s)},t_0)\right], \tag{1.10B}$$

where

$$t_k^{(O)} = t_k^{(E)}.$$

Case 3: The Epoch, $t_0^{(s)}$, is Unknown

The solution for $t_0^{(s)}$ is the most laborious of the three possible cases. Since, given experimental data on the times of marker transmissions, it is most important to obtain an accurate estimate of the epoch, $t_0^{(s)}$, this case is discussed in more detail. The detailed procedures presented here are those that have been programmed by The Applied Physics Laboratory and have proved to be adequate.

Clearly, if an approximate value for the epoch is known, an iterative procedure analogous to the one given for Case 2 can be used. Furthermore, even if the starting value for $t_0^{(s)}$ in the iterative procedure is grossly in error, the contribution of the sums over the functions, $\{v_\ell(t,t_0)\}$, is so small that the iterative procedure still converges rapidly. Because of this, it has been found that obtaining an accurate starting value for $t_0^{(s)}$, so that the iterative procedure is efficient, is not the principal problem. The greatest difficulty is in determining which data is spurious so that only valid data is used in the final computation for $t_0^{(s)}$. Because of this, the obvious procedure of choosing the epoch to be near one of the experimental data points does not necessarily decrease computing time.

It is very convenient to choose the epoch, $t_0^{(s)}$, to be close to the epoch for the frequency equation. In the APL programs, the epoch for the frequency equation is chosen to be identical to the epoch for the satellite orbit. Using this epoch as a starting value for the procedure to determine $t_0^{(s)}$ insures that all epochs remain near each other in time - an obvious advantage from the standpoint of "bookkeeping".

In detail, the APL prescription for determining the epoch, $t_0^{(s)}$ is the following. Given,

$$t_k^{(E)},\ k = 1, 2, \ldots\ldots n_T = \text{experimental time data},$$

$$w_k,\ k = 1, 2, \ldots n_T = \text{a priori statistical wts},$$

$$t_0, \qquad\qquad = \text{epoch for frequency equation}.$$

Set:

$$t_0^{(s,0)} = t_0$$

where $t_0^{(s,m)}$ is the m-th iterative result for $t_0^{(s)}$. For every data point, compute

$$x_k^{(0)} = \frac{f_{S,0}}{N_{C,S}} \left(t_k^{(E)} - t_0^{(s,0)}\right)$$

$$+ \frac{1}{N_{C,S}} \sum_{\ell=1}^{n_f} a_{f,\ell} \left[v_\ell(t_k^{(E)}, t_0) - v_\ell(t_0, t_0)\right],$$

$$K_k^{(0)} = \text{Int}\left[x_k^{(0)}\right]. \tag{1.11A}$$

Then, for every data point, compute:

$$t_0^{(s,1)} = t_0^{(s,0)} + \frac{N_{C,S}}{f_{S,0}} \sum_{k=\ell}^{n_T} w_k(x_k^0 - K_k^0)$$

$$\sigma_1^2 = \sum_{k=1}^{n_T} w_k[t_0^{(s,1)} - t_0^{(s,0)} - \frac{N_{C,S}}{f_{S,0}} (x_k^{(0)} - K_k^{(0)})]^2. \qquad (1.11B)$$

This equation provides a good starting value for $t_0^{(s)}$, providing that only a small percentage of the time data points are spurious. The normal iteration loop is given as follows.

Compute just once more, for each data point,

$$K_k = \mathrm{Int}\left[\frac{f_{S,0}}{N_{C,S}} (t_k^{(E)} - t_0^{(s,0)}) + \frac{1}{N_{C,S}} \sum_{\ell=1}^{n_f} a_{f,\ell}(v_\ell(t_k^{(E)}, t_0) - v_\ell(t_0^{(s,1)},t_0))\right], \qquad (1.12A)$$

$k = 1, 2, \ldots .n_T$,

and then, iterate with each K_k fixed,

$$\Delta t_{0,k}^{(s,m)} = t_k^{(E)} - t_0^{(s,m)} - \frac{N_{C,S}}{f_{S,0}} K_k$$

$$+ \frac{1}{f_{S,0}} \sum_{\ell=1}^{n_f} a_{f,\ell}(v_\ell(t_k^{(E)},t_0) - v_\ell(t_0^{(s,m)},t_0)), \qquad (1.12B_1)$$

$$\Delta t_0^{(s,m)} = \frac{\sum\limits_{k=1}^{n_T}{}' \; w_k \; \Delta t_{0,k}^{(s,m)}}{\sum\limits_{k=1}^{n_T}{}' \; w_k}, \tag{1.12B$_2$}$$

$$\sigma_{m+1}^2 = \frac{\sum\limits_{k=1}^{n_T}{}' \; w_k \; [\Delta t_{0,k}^{(s,m)}]^2}{\sum\limits_{k=1}^{n_T}{}' \; w_k}, \tag{1.12B$_3$}$$

where the primed sum includes only those data points for which

$$[\Delta t_{0,k}^{(s,m)}]^2 \leq 9 \; \sigma_m^2 \; , \tag{1.12C}$$

and

$$t_0^{(s,m+1)} = t_0^{(s,m)} + \Delta t_0^{(s,m)} \; . \tag{1.12D}$$

Breakout of the iteration loop occurs when $\Delta t_0^{(s,m)}$ becomes less than 10^{-5} seconds and the number of valid data points has remained constant for two iterations.

Having inferred a value for the epoch, $t_0^{(s)}$, any experimental data point can be checked by using the procedures shown in Cases 1 and 2. Clearly, only data originating from 'time standard stations' should be used in evaluating the epoch $t_0^{(s)}$.

The procedures of Case 2 can be used to compute an ephemeris of times when time markers are transmitted and can be used to extrapolate time marker transmission times into the future. However, for transmittal of time ephemeris data from one agency to another, the following simplified equation for providing time marker transmission times is convenient.

The form of the equation is chosen to be

$$t_k = t_0^{(s)} + C_1 K + C_2 K^2$$

where the constants C_1 and C_2 are chosen such that the t_k as computed by this equation agrees to sufficient accuracy with the more accurate equations given previously. Experimental data from 1961 $\alpha\eta$1 indicates that for oscillators under active temperature control, an equation of this form will remain valid for over one week to an accuracy greater than 10 milliseconds. Consequently, such an equation is ideal for station alerting purposes.

II. RECOGNITION AND PRE-PROCESSING OF SATELLITE TIME DATA

Satellite time data for any one satellite pass, as punched on paper tape and received at the Satellite Control Center, is interspersed with the doppler data for the same satellite pass. This section presents the information required to differentiate between doppler and time data points, and to reduce the experimental time data to experimental times, $t_k^{(E)}$, when the appropriate satellite transmitted satellite time markers.*

*This report does not consider the change in data format necessitated when 7 digits instead of 6 are used for the doppler period count.

A typical doppler data point, when punched on TWX paper tape, has the format:

CL XXXXX YYYYYY

where

C = TWX carriage return

L = TWX line-feed

XXXXX = t_μ

= First 5 TWX characters (integers) denoting the station time of the μ-th doppler data point, (sec).

YYYYYY = $T\mu$

= Last 6 TWX characters (integers) denoting the doppler period count, (microseconds).

When a station is taking satellite time data, any arbitrary doppler data point can be suppressed and replaced by a satellite time data point. The format of the satellite time data point can be in either of two different formats, depending upon the type of timing equipment installed in the statio. For identification purposes, these two formats will be denoted as APL or NOTS format. They are:

APL Format	NOTS Format
CL 9ZZZZ XXXXX0	CL XXXXX 00ZZZZ

where

C = TWX carriage return

L = TWX line-feed

XXXXX = t_μ

(Format cont'd.)

= 5 TWX characters (integers) giving the integer part when the satellite time mark is received, (sec).

ZZZZ = ΔT_μ

= 4 TWX characters (integers) giving one correction to be applied to t_μ (tenths of milliseconds).

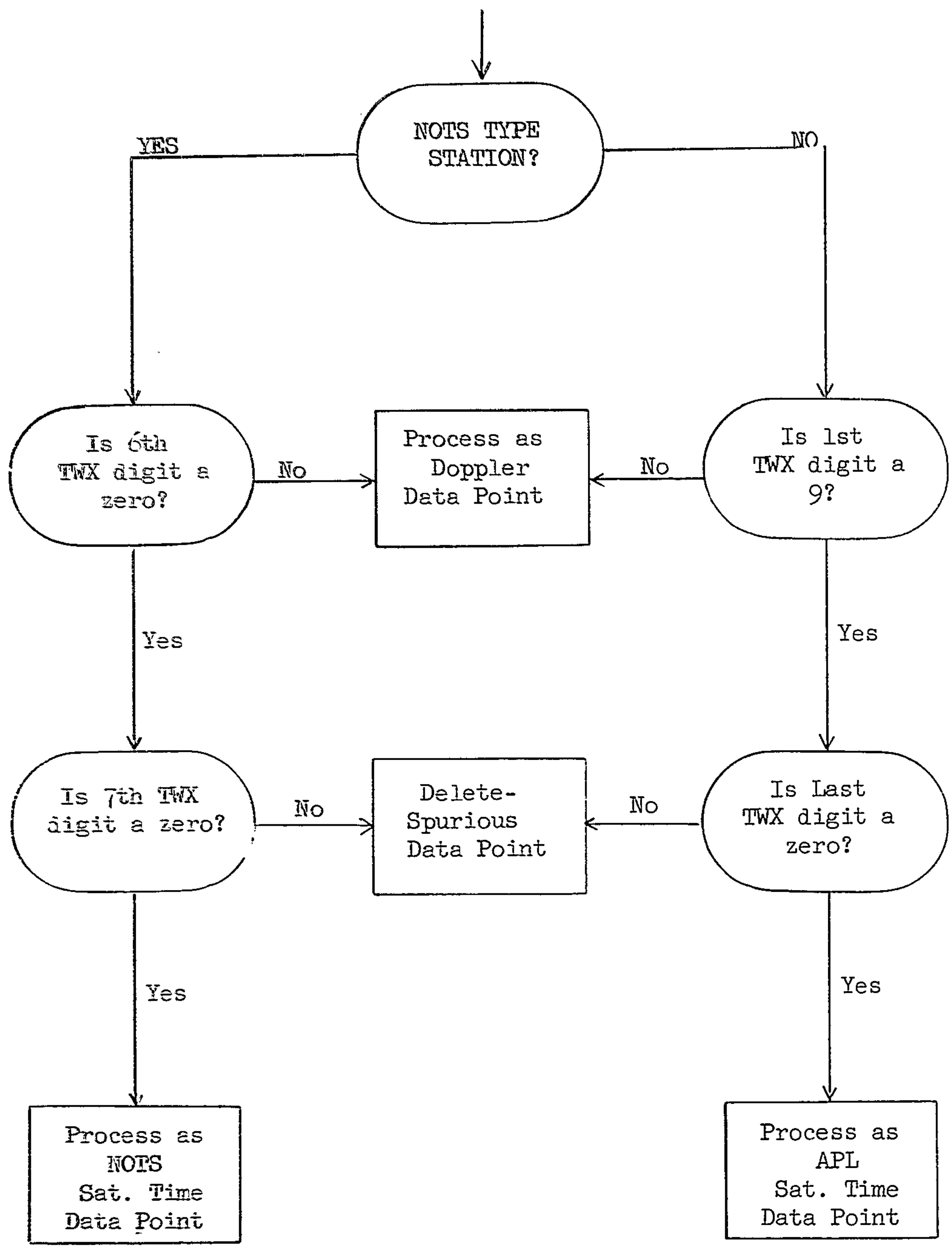
With either format a satellite time data point can be uniquely identified when interspersed with doppler data because the TWX 9 in the APL format and the first TWX 0 (zero) in the NOTS format are illegal characters for a doppler point. The following flow chart is an example of a testing network to correctly identify each data point.

Upon identification of a satellite time data point, the TWX C and L should be deleted, the remaining 11 TWX digits appropriately unpacked, and transformed from BCI to the two floating point numbers.

t_μ = integer part of satellite time in seconds

ΔT_μ = correction for satellite time in units of 10^{-4} seconds.

The reconstruction of the time (WWV) that the satellite transmitted a marker is given below. This reconstruction also depends upon the type of station equipment (APL or NOTS) and, if obtained from APL type equipment, further depends upon the satellite transmitting the time marker.


NOTS TYPE STATION?
YES
NO
Is 6th TWX digit a zero?
No
Process as Doppler Data Point
No
Is 1st TWX digit a 9?
Yes
Yes
Is 7th TWX digit a zero?
No
Delete-Spurious Data Point
No
Is Last TWX digit a zero?
Yes
Yes
Process as NOTS Sat. Time Data Point
Process as APL Sat. Time Data Point

A. Reconstruction of APL Type Time.

$$t_k^{(E)} = [t_\mu - 1 + 10^{-4}\, \Delta T_\mu - t_D^{(s)}] + \Delta t_H - \Delta t. \tag{2.1}$$

Where

$t_k^{(E)}$ = time (station's estimate of WWV time) at which time marker transmitted (sec),

$t_\mu, \Delta T_\mu$ = defined above,

$t_D^{(s)}$ = delay time in station time gear for the s-th satellite (sec),

Δt_H = station clock correction given in data header (sec),

Δt_{TR} = transmission time from satellite to station (sec),

and where;

Satellite	Value of t_D^s (sec)
1961 ο1	3.27692×10^{-2}
1961 αη1	1.66406×10^{-2}
1962 βμ1	0.0

B. Reconstruction of NOTS Type Time

$$t_k^{(E)} = [t_\mu + 1 - 10^{-4}\, \Delta T_\mu - \Delta t_D] + \Delta t_H - \Delta t_{TR} \tag{2.2}$$

where

$t_k^{(E)}$ = time (station's estimate of WWV time) at which time marker transmitted by satellite, (sec),

t_μ, ΔT_μ = defined above,

Δt_H = station clock correction given in data header, (sec),

Δt_D = NOTS station delay time

Δt_{TR} = transmission time from satellite to station, (sec).

Equations (2.1) and (2.2) define $t_k^{(E)}$ which is the stations best estimate of the time (WWV) that a satellite time mark is transmitted from the satellite. Typically, from one to fifty such times can be received during a single pass. Currently t_D for the NOTS type time is zero.

The Johns Hopkins University
APPLIED PHYSICS LABORATORY
Silver Spring, Maryland

Initial distribution of this document has been made in accordance with a list on file in the Technical Reports Group of The Johns Hopkins University, Applied Physics Laboratory.